Assignment 2, Question 5. Solution :

Let A be the event that elder child is a girl, B be the event that younger child is a girl and C be the event that random child is a girl.

Then by assumptions given in the question,

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(C) = \frac{1}{2}$$

(Upon randomly chose one child out of two children, the child will be a girl with prob. $\frac{1}{2}$).

We need to find $P(A \cap B \mid C)$ i.e. P(Both are girls | random child is a girl).

By def^n of conditional probability,

$$P(A \cap B \mid C) = \frac{P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(A \cap B)}{P(C)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}.$$

Note that $P(A \cap B \cap C) = P(A \cap B) = P(\text{Both are girls})$

If A & B both happen, then C is guaranteed to happen.